REAL OPTION VALUE

CHAPTER 14 INCENTIVE OPTIONS

12 March 2016

Incentive options can be viewed using the toolkit implicit in previous chapters of real payoff diagrams, entry and exit options, and perpetual American puts and calls. Incentive options may be granted (or required by) governments to encourage early investment in "desirable" projects such as renewable energy facilities, infrastructure investments like roads, bridges and other transportation, and in general public-private partnerships governing new facilities like schools, hospitals, and recreation areas.

These incentive options are classified as (i) proportional revenue (or price and/or quantity) subsidies, where the market price and/or the quantity of production is uncertain or low, but the subsidy is proportional to the quantity produced (ii) supplementary revenue (or price and/or quantity) subsidies, where the market price and/or the quantity of production and/or the exogenous subsidy is uncertain (iii) revenue floors and ceilings, where the subsidy is related over time to the actual quantities produced or market prices. Examples of (i) are so-called Feed-in-tariffs fixed amount subsidies per unit production, (ii) renewable "green" certificates, which have an uncertain value but are usually allocated per unit of production, and (iii) government minimum revenue guarantees, sometimes accompanied by maximum revenue ceilings.

In addition, governments provide incentives for free or at low cost (sport stadiums, concessions, priority access, protection through tariffs, quotas or security) in order to encourage "desirable" activities, or investment cost reliefs, consisting of direct grants and soft loans, tax credits or excess depreciation, which are not directly considered here, except in examining sensitivities of thresholds and real option value to changes in investment costs or taxation. Some of these incentives can also be characterized as incentive options. Most of these incentives can be evaluated in terms of the real option value compared to that paid to the government (taxes, concession and user fees and royalties) weighted against the immediate or eventual cost for the government. Also it is interesting to study the effect on the real option value, and on the threshold that justifies immediate investment, of price, quantity and subsidy changes. Who gets/gives what, when, how, and why are almost always critical considerations in incentive options.

14.1 Proportional Subsidies

This section considers a menu of possible arrangements, that is some characteristic subsidies for such facilities, first where there is no subsidy (Model 1); then assuming there is a permanent subsidy proportional to the revenue (Model 2); finally assuming there is a retractable subsidy proportional to the revenue (Model 3), as suggested in the Adkins and Paxson (2014), Appendix.

Proportional Stochastic Revenue Models

Consider a perpetual opportunity to construct an electricity generating facility producing Q MWhrs/pa, using solar power, at a fixed investment cost K. This investment cost is treated as irreversible or irrecoverable once incurred. The value of this investment opportunity, denoted by ROV, depends on the amount of output Q, and the price per unit of output, denoted by P, P*Q=R, revenue. R is assumed to be stochastic and to follow a geometric Brownian motion process:

$$\mathrm{dR} = \theta_R \mathrm{R}\mathrm{d}t + \sigma_R R \mathrm{d}Z \tag{1}$$

where θ_R denotes the instantaneous risk neutral drift parameter (equals δ the asset yield), σ_R the instantaneous volatility, and $\mathrm{d}Z$ the standard Wiener process. The differential equation representing the value to invest for an inactive investor with an appropriate investment opportunity (based perhaps on approval for the facility or a concession for infrastructure) is:

$$\frac{1}{2}\sigma_R^2 R^2 \frac{\partial^2 ROV_1}{\partial R^2} + \theta_R R \frac{\partial ROV_1}{\partial R} - rROV_1 = 0. \tag{2}$$

where r is the risk-free rate. Adkins and Paxson (2014) show that the solution to (2) is:

$$ROV_1 = B_1 R^{\beta_1}. \tag{3}$$

β_1 is the power parameter for this option value function. Since there is an incentive to invest when R is sufficiently high but a disincentive when sufficiently low, the power parameter value is positive. Also, the power parameter is determined using the characteristic root equation (which is the positive root of a simple quadratic equation) found by substituting (3) in (2):

$$\beta_1 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}\,. \tag{4}$$

After the investment, the solar plant generates revenue equaling (1+τ)*R, where τ is the permanent subsidy proportional to the revenue sold (τ=0 indicates no possible subsidy). So from (2), the valuation relationship for the operational state is:

$$\frac{1}{2}\sigma_R^2 R^2 \frac{\partial^2 ROV_1}{\partial R^2} + \theta_R R \frac{\partial ROV_1}{\partial R} + (1+\tau)R - rROV_1 = 0. \tag{5}$$

After the investment (K), the solution to (5) is:

$$\frac{(1+\tau)R}{r-\theta_R}\,. \tag{6}$$

Model 1

The subsidy is set to equal zero in Model 1. If the threshold revenue signaling an optimal investment is denoted by $\hat{R}_1$, then:

$$\hat{R}_1 = \frac{\beta_1}{\beta_1 - 1} K\left(r-\theta_R\right). \tag{7}$$

The value for the investment opportunity is defined by:

$$ROV_1 = \begin{cases} B_1 R^{\beta_1} \quad \text{for} \quad R < \hat{R}_1, \\ \dfrac{R}{r-\theta_R} - K \quad \text{for} \quad R \geq \hat{R}_1. \end{cases} \tag{8}$$

where:
$$B_1 = \frac{\hat{R}_1^{1-\beta_1}}{\beta_1 (r-\theta_R)}. \tag{9}$$

Model 2

For a positive proportional permanent subsidy τ , the corresponding results are:

$$\hat{R}_2 = \frac{\beta_1}{\beta_1 - 1} K \frac{(r-\theta_R)}{(1+\tau\ )}, \tag{10}$$

$$ROV_2 = \begin{cases} B_2 R^{\beta_1} \quad \text{for} \quad R < \hat{R}_2, \\ \dfrac{R(1+\tau\ )}{r-\theta_R} - K \quad \text{for} \quad R \geq \hat{R}_2, \end{cases} \tag{11}$$

$$B_2 = \frac{(1+\tau\ )\hat{R}_2^{1-\beta_1}}{\beta_1 (r-\theta_R)} \tag{12}$$

Model 3A

The probability of a sudden unexpected withdrawal of the subsidy is denoted by λ. If the revenue threshold signaling an optimal investment is denoted by $\hat{R}_3$, then its solution is found implicitly from:

$$\hat{R}_3 = \frac{\beta_3}{\beta_3 - 1} K \frac{r-\theta_R}{1+(1-\lambda)\tau} + B_1 \hat{R}_3^{\beta_1} \frac{\beta_3 - \beta_1}{\beta_3 - 1} \tag{13}$$

where B_1 is from (9). The value for the investment opportunity is specified by:

$$ROV_3 = \begin{cases} B_3 R^{\beta_3} + B_1 R^{\beta_1} \text{ for } R < \hat{R}_3, \\ \dfrac{R\left(1+\left(1-\lambda\right)\tau\ \right)}{r-\theta_R} - K \text{ for } R \geq \hat{R}_3, \end{cases} \tag{14}$$

where: $$B_3 = \frac{(1+\left(1-\lambda\right)\tau_M)\hat{R}_3^{1-\beta_3}}{\beta_3(r-\theta_R)} - \frac{\beta_1}{\beta_3} B_1 \hat{R}_3^{\beta_1-\beta_3}. \tag{15}$$

β_3 is the positive root of (4) with λ added to r. For $\lambda = 0$, when there is no likelihood of the subsidy being withdrawn unexpectedly, $\beta_3 = \beta_1$ and Model 3 simplifies to the Model 2 solution. It is easy to put these formulae into Excel as shown in Figures 1, 2, 3 below.

Model 3B

The probability of a sudden unexpected introduction of a permanent subsidy is denoted by λ. If the revenue threshold signaling an optimal investment is denoted by $\hat{R}_4$, then its solution is found implicitly

from: $$\hat{R}_3 = \frac{\beta_3}{\beta_3 - 1} \frac{r-\theta_R}{1+\lambda\tau} (K + \frac{\lambda}{r+\lambda} B_2 \hat{R}_2^{\beta_1}) \tag{16}$$

where B_2 is from (12). The value for the investment opportunity is specified by:

$$ROV_4 = \begin{cases} B_4 R^{\beta_3} + \dfrac{\lambda}{r+\lambda} B_2 R^{\beta_1} \text{ for } R < \hat{R}_4, \\ \dfrac{R\left(1+\lambda\tau\ \right)}{r-\theta_R} - K \text{ for } R \geq \hat{R}_4, \end{cases} \tag{17}$$

where: $$B_3 = \frac{(1+\lambda\tau\ )\hat{R}_4^{1-\beta_3}}{\beta_3(r-\theta_R)}. \tag{18}$$

For $\lambda = 0$, when there is no likelihood of an unexpected introduction of a permanent proportional subsidy, Model 3B simplifies to the Model 1 solution.

Figure 1

	A	B	C	D
1	**REVENUE MODEL 1**			
2	INPUT	Stochastic R		
3	P	22.50	Per MWhr	
4	Q	10.00	MWhrs/per annum	
5	R	225.00	B3*B4	
6	K	4000.00	Per Capacity of 10 MWhrs/per annum	
7	σ	0.20	Template	
8	r	0.08	Given	
9	θ	0.04	Template	
10	τ	0.00	NO SUBSIDY	
11	$r-\theta$	0.04	B8-B9	
12	λ	0.00	Probability	
13	OUTPUT			
14	ROV1	2456.34	IF(B5<B18,B17*(B5^B16),B15)	
15	V-K	1625.00	((1+B10)*B5/B11)-B6	
16	β_1	1.5616		
17	B1	0.5215	(B18^(1-B16))/(B16*B11)	
18	R*	444.92	B6*B11*(B16/(B16-1))	
19	β_1	(1/B7^2)*(-(B11-0.5*(B7^2))+SQRT((B11-0.5*(B7^2))^2+(2*B8)*(B7^2)))		

Figure 2 illustrates a subsidy of τ=1, which results in a threshold R*=R, justifying immediate investment.

Figure 3 shows that when the probability of subsidy withdrawal is zero, Model 3A is reduced to Model 2 in Figure 2.

Figure 4A shows Model 3A with a positive probability of withdrawal, which reduces R* significantly, a "flighty bird in hand" motivates early investment.

Figure 2

	A	B	C	D
1	**REVENUE MODEL 2**			
2	INPUT	Stochastic R		
3	P	22.50		
4	Q	10.00		
5	R	225.00	B3*B4	
6	K	4000.00		
7	σ	0.20		
8	r	0.08		
9	θ	0.04		
10	τ	1.00		
11	$r-\theta$	0.04	B8-B9	
12	λ	0.00	Probability	
13	OUTPUT			
14	ROV2	7250.00	IF(B5<B18,B17*(B5^B16),B15)	
15	V-K	7250.00	((1+B10)*B5/B11)-B6	
16	β_1	1.5616		
17	B2	1.5392	((1+B10)*B18^(1-B16))/(B16*B11)	
18	R*	222.46	(B6*B11/(1+B10))*(B16/(B16-1))	
19	β_1	(1/B7^2)*(-(B11-0.5*(B7^2))+SQRT((B11-0.5*(B7^2))^2+(2*B8)*(B7^2)))		

Figure 3

	A	B	C	D	E
1	**REVENUE MODEL 3**				
2	INPUT	Stochastic R			
3	P	22.50			
4	Q	10.00			
5	R	225.00	B3*B4		
6	K	4000.00			
7	σ	0.20			
8	r	0.08			
9	θ	0.04			
10	τ	1.00			
11	$r-\theta$	0.04	B8-B9		
12	λ	0.00	Probability		
13	OUTPUT				
14	ROV3	7250.00	IF(B5<B18,B17*(B5^B16)+B24*(B5^B23),B15)	14	
15	V-K	7250.00	((1+(1-B12)*B10)*B5/B11)-B6		
16	$\beta3$	1.5616		4	
17	B3	1.0178		15	
18	R*3	**222.46**			
19	Solver	0.0000	Set B19=0, Changing B18	13	
20	$\beta1$	1.5616			
21	B1	0.5215			
22	R*1	444.92			
23	$\beta3$	(1/B7^2)*(-(B11+B12-0.5*(B7^2))+SQRT((B11+B12-0.5*(B7^2))^2+(2*(B8+B12)*(B7^2))))			
24	R*3	((B6*B11)/(1+(1-B12)*B10))*(B16/(B16-1))+B24*(B18^B23)*((B16-B23)/(B16-1))-B18			
25	B3	((1+(1-B12)*B10)*B18^(1-B16))/(B16*B11)-(B23/B16)*B24*(B18^(B23-B16))			

Figure 4A

	A	B	C	D	E
1	**REVENUE MODEL 3A**				
2	INPUT	Stochastic R			
3	P	22.50			
4	Q	10.00			
5	R	225.00	B3*B4		
6	K	4000.00			
7	σ	0.20			
8	r	0.08			
9	θ	0.04			
10	τ	1.00			
11	$r-\theta$	0.04	B8-B9		
12	λ	0.10	Probability		
13	OUTPUT				
14	ROV3	6687.50	IF(B5<B18,B17*(B5^B16)+B24*(B5^B23),B15)		
15	V-K	6687.50	((1+(1-B12)*B10)*B5/B11)-B6		
16	$\beta3$	1.2426			
17	B3	11.9792			
18	R*3	**56.65**			
19	Solver	0.0000	Set B19=0, Changing B18		
20	$\beta1$	1.5616			
21	B1	0.5215			
22	R*1	444.92			
23	$\beta3$	(1/B7^2)*(-(B9+B12-0.5*(B7^2))+SQRT((B9+B12-0.5*(B7^2))^2+(2*(B8+B12)*(B7^2))))			
24	R*3	((B6*B11)/(1+(1-B12)*B10))*(B16/(B16-1))+B24*(B18^B23)*((B16-B23)/(B16-1))-B18			
25	B3	((1+(1-B12)*B10)*B18^(1-B16))/(B16*B11)-(B23/B16)*B24*(B18^(B23-B16))			

	A	B	C	D	E
1	**REVENUE MODEL 3**				
2	INPUT	Stochastic R			
3	P	22.50			
4	Q	10.00			
5	R	225.00	B3*B4		
6	K	4000.00			
7	σ	0.20			
8	r	0.08			
9	θ	0.04			
10	τ	1.00			
11	r–θ	0.04	B8-B9		
12	λ	0.10	Probability		
13	OUTPUT				
14	ROV3	6687.50	IF(B5<B18,B17*(B5^B16)+B24*(B5^B23),B15)	14	
15	V-K	6687.50	((1+(1-B12)*B10)*B5/B11)-B6		
16	β3	1.2426		4	
17	B3	11.9792		15	
18	R*3	**56.65**			
19	Solver	0.0000	Set B19=0, Changing B18	13	
20	β1	1.5616			
21	B1	0.5215			
22	R*1	444.92			
23	β3	(1/B7^2)*(-(B11+B12-0.5*(B7^2))+SQRT((B11+B12-0.5*(B7^2))^2+(2*(B8+B12)*(B7^2))))			
24	R*3	((B6*B11)/(1+(1-B12)*B10))*(B16/(B16-1))+B24*(B18^B23)*((B16-B23)/(B16-1))-B18			
25	B3	((1+(1-B12)*B10)*B18^(1-B16))/(B16*B11)-(B23/B16)*B24*(B18^(B23-B16))			

Figure 4B

	A	B	C	D
1			**REVENUE MODEL 3B**	
2	INPUT	Stochastic R		
3	P	22.50		
4	Q	10.00		
5	R	225.00	B3*B4	
6	K	4000.00		
7	σ	0.20		
8	r	0.08		
9	θ	0.04		
10	τ	1.00		
11	$r-\theta$	0.04	B8-B9	
12	λ	0.10	Probability	
13	OUTPUT			
14	ROV4	7108.92	IF(B5<B18,B17*(B5^B16)+(B12/(B8+B12))*B22*(B23^B21),B15)	
15	V-K	2187.50	(1+B12*B10)*B5/B11-B6	
16	$\beta 3$	1.2426		
17	B4	3.7637	((1+B10*B12)*B18^(1-B16))/(B16*B11)	
18	R*4	1481.88		
19	$\beta 3$	(1/B7^2)*(-(B11+B12-0.5*(B7^2))+SQRT((B11+B12-0.5*(B7^2))^2+(2*(B8+B12)*(B7^2))))		
20	R*4	(B16/(B16-1))*(B11/(1+B10*B12))*(B6+(B12/(B8+B12))*B22*(B23^B21))		
21	$\beta 1$	1.5616		
22	B2	1.5392		
23	R*2	222.4621		

Figure 4B shows Model 3B with a positive probability of permanent subsidy that cannot be withdrawn, which increases R* significantly, as investors presumed to have a proprietary option to invest await for the desired benefit, deferring investment.

14.2 Exogenous Subsidies

Model 4 Stochastic Price, Subsidy and Quantity

Now consider a perpetual opportunity to construct a renewable energy facility at a fixed investment cost K, where the subsidy is exogenous like a "green certificate". The value of this investment opportunity, denoted by F_1, depends on the amount of output sold per unit of time, denoted by Q, the market price per unit of output, denoted by P, and the subsidy per output unit, S. In the general model, all of these variables are assumed to be stochastic and are assumed to follow geometric Brownian motion processes (gBm):

$$\mathrm{d}X = \theta_X X\mathrm{d}t + \sigma_X X\mathrm{d}Z \tag{1}$$

for $X \in \{P, S, Q\}$, where θ denotes the risk neutral instantaneous drift parameter, σ the instantaneous volatility, and $\mathrm{d}Z$ the standard Wiener process. Potential correlation between the variables is represented by ρ.

The partial differential equation (PDE) representing the value to invest for an inactive firm with an appropriate perpetual investment opportunity (based on perhaps approval for the facility or a concession for infrastructure) is:

$$\begin{aligned}
&\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_1}{\partial P^2} + \frac{1}{2}\sigma_Q^2 Q^2 \frac{\partial^2 F_1}{\partial Q^2} + \frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 F_1}{\partial S^2} \\
&+PQ\rho_{PQ}\sigma_P\sigma_Q \frac{\partial^2 F_1}{\partial P \partial Q} + PS\rho_{PS}\sigma_P\sigma_S \frac{\partial^2 F_1}{\partial P \partial S} + QS\rho_{QS}\sigma_Q\sigma_S \frac{\partial^2 F_1}{\partial Q \partial S} \\
&+\theta_P P \frac{\partial F_1}{\partial P} + \theta_Q Q \frac{\partial F_1}{\partial Q} + \theta_S S \frac{\partial F_1}{\partial S} - rF_1 = 0.
\end{aligned} \tag{2}$$

where r is the risk-free rate. Following Adkins and Paxson (2016), when P,Q, or S are below $\hat{P}, \hat{Q}, \hat{S}$ that justify immediate investment, the solution to (2) is:

$$ROV_1 = F_1 = A_1 P^{\beta_1} Q^{\gamma_1} S^{\eta_1}. \tag{3}$$

where β_1, γ_1 and η_1 are the power parameters for this option value function. Since there is an incentive to invest when P, Q and S are sufficiently high but a disincentive when these are sufficiently low, we expect that all power parameter values are positive. Also, the parameters are linked through the characteristic root equation found by substituting (3) in (2):

$$\begin{aligned}
&Q(\beta_1, \gamma_1, \eta_1) = \tfrac{1}{2}\sigma_P^2\beta_1(\beta_1 - 1) + \tfrac{1}{2}\sigma_Q^2\gamma_1(\gamma_1 - 1) + \tfrac{1}{2}\sigma_S^2\eta_1(\eta_1 - 1) + \\
&\rho_{PQ}\sigma_P\sigma_Q\beta_1\gamma_1 + \rho_{PS}\sigma_P\sigma_S\beta_1\eta_1 + \rho_{QS}\sigma_Q\sigma_S\gamma_1\eta_1 \\
&+\theta_P\beta_1 + \theta_Q\gamma_1 + \theta_S\eta_1 - r = 0
\end{aligned}. \tag{4}$$

After the investment, the plant generates revenue equaling $PQ + SQ$, with the present value factor of parts of this net revenue denoted $k_{P,}$ k_Q and k_S (no operating costs or taxes) (life assumed to be T=20 years in the base case)[1].

[1] This is the methodology in Boomsma and Linnerud (2015).

$$k_P = \frac{1-e^{-(r-\theta_P)*T}}{(r-\theta_P)}, k_{PQ} = \frac{1-e^{-(r-\theta_P-\theta_Q)*T}}{(r-\theta_P-\theta_Q)} \tag{5}$$

$$k_Q = \frac{1-e^{-(r-\theta_Q)*T}}{(r-\theta_Q)} \tag{6}$$

$$k_S = \frac{1-e^{-(r-\theta_S)*T}}{(r-\theta_S)}, k_{SQ} = \frac{1-e^{-(r-\theta_S-\theta_Q)*T}}{(r-\theta_S-\theta_Q)}, \tag{7}$$

The value matching relationship, when the real option value upon exercise is equal to the net present value of the investment (NPV), is:

$$A_1\hat{P}^{\beta_1}\hat{Q}^{\gamma_1}\hat{S}_1^{\eta_1} = k_{PQ}\hat{P}\hat{Q} + k_{SQ}\hat{S}_1\hat{Q} - K \tag{8}$$

The three associated smooth pasting conditions can be expressed as:

$$\beta_1 A_1\hat{P}^{\beta_1}\hat{Q}^{\gamma_1}\hat{S}_1^{\eta_1} = k_{PQ}\hat{P}\hat{Q} \tag{9}$$

$$\gamma_1 A_1\hat{P}^{\beta_1}\hat{Q}^{\gamma_1}\hat{S}_1^{\eta_1} = k_{PQ}\hat{P}\hat{Q} + k_{SQ}\hat{S}_1\hat{Q} \tag{10}$$

$$\eta_1 A_1\hat{P}^{\beta_1}\hat{Q}^{\gamma_1}\hat{S}_1^{\eta_1} = k_{SQ}\hat{S}_1\hat{Q} \tag{11}$$

A quasi-analytical solution to the set of five equations 4-8-9-10-11 for 7 unknowns

$\hat{P}, \hat{Q}, \hat{S}_1, \beta_1, \gamma_1, \eta_1, A_1$ is obtained by assuming $\hat{P} = P, \hat{Q} = Q$ as in Adkins and Paxson (2016), and then finding $\hat{S}_1, \beta_1, \gamma_1, \eta_1, A_1$. An analytical solution is obtained by recognizing that:

$$A_1 = k_{PQ}\hat{P}\hat{Q} / \beta_1\hat{P}^{\beta_1}\hat{Q}^{\gamma_1}\hat{S}_1^{\eta_1} \tag{12}$$

and

$$\hat{S}_1 = \eta_1 k_{PQ}\hat{P} / \beta_1 k_{SQ} \tag{13}$$

This implies that $$\gamma_1 = \beta_1 + \eta_1 \tag{14}$$

Eliminating A_1 from (8) yields:

$$\beta_1 = k_{PQ}\hat{P}\hat{Q} / (k_{PQ}\hat{P}\hat{Q} + k_{SQ}\hat{S}_1\hat{Q} - K) \tag{15}$$

So $$\eta_1 = 1 + \beta_1\left(\frac{K}{k_{PQ}\hat{P}\hat{Q}} - 1\right) \tag{16}$$

Eliminating γ_1 and η_1 from the characteristic root equation (4) yields the quadratic equation:

$$Q(\beta_1) = \beta_1^2\{a\} + \beta_1\{b\} - \{c\} = 0 \tag{17}$$

$$\begin{aligned} a = \Big\{ & \tfrac{1}{2}\sigma_P^2 - \rho_{PS}\sigma_P\sigma_S + \tfrac{1}{2}\sigma_S^2 \\ & + \frac{K^2}{2\hat{P}^2\hat{Q}^2 k_{PQ}^2}[\sigma_Q^2 + 2\rho_{QS}\sigma_Q\sigma_S + \sigma_S^2] \\ & + \frac{K}{\hat{P}\hat{Q}k_{PQ}}[\rho_{PQ}\sigma_P\sigma_Q + \rho_{PS}\sigma_P\sigma_S - \rho_{QS}\sigma_Q\sigma_S - \sigma_S^2] \ \Big\} \end{aligned}$$

$$\begin{aligned} b = \Big\{ & \theta_P - \theta_S - \tfrac{1}{2}\sigma_P^2 - \tfrac{1}{2}\sigma_S^2 + \rho_{PQ}\sigma_P\sigma_Q + \rho_{PS}\sigma_P\sigma_S - \rho_{QS}\sigma_Q\sigma_S \\ & + \frac{K}{\hat{P}\hat{Q}k_{PQ}}[\theta_Q + \theta_S + \frac{\sigma_Q^2}{2} + 2\rho_{QS}\sigma_Q\sigma_S + \frac{\sigma_S^2}{2}]\Big\} \end{aligned}$$

$$c = -\left\{ r - \theta_Q - \theta_S - \rho_{QS}\sigma_Q\sigma_S \right\}$$

This equation has the simple quadratic solution:

$$\beta_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{18}$$

Model 5

Stochastic Price and Subsidy with a Deterministic Quantity

We now modify the analysis to consider the impact on the investment decision of a permanent but uncertain government subsidy, denoted by S, but where the output Q sold per unit of time is deterministic.

The PDE is:

$$\begin{aligned}&\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_2}{\partial P^2} + \frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 F_2}{\partial S^2} \\ &+PS\rho_{PS}\sigma_P\sigma_S \frac{\partial^2 F_2}{\partial P \partial S} + \theta_P P \frac{\partial F_2}{\partial P} + \theta_Q Q \frac{\partial F_2}{\partial Q} + \theta_S S \frac{\partial F_2}{\partial S} - rF_2 = 0.\end{aligned} \tag{19}$$

where θ_X denote the risk-neutral drift rates and r the risk-free rate, (θ=r-δ). The solution to (19) is:

$$ROV_2 = F_2 = A_2 P^{\beta_2} Q^{\gamma_2} S^{\eta_2}. \tag{20}$$

where β_2 , γ_2 and η_2 are the power parameters for this option value function (allowing for a deterministic quantity). We expect that all power parameter values are positive. Also, the parameters are linked through the characteristic root equation found by substituting (20) in (19):

$$\begin{aligned}&Q(\beta_2,\gamma_2,\eta_2) = \tfrac{1}{2}\sigma_P^2\beta_2(\beta_2-1) + \tfrac{1}{2}\sigma_S^2\eta_2(\eta_2-1) + \\ &+\rho_{PS}\sigma_P\sigma_S\beta_2\eta_2 + \theta_P\beta_2 + \theta_Q\gamma_2 + \theta_S\eta_2 - r = 0 \qquad .\end{aligned} \tag{21}$$

The value matching relationship becomes:

$$A_2 \hat{P}^{\beta_2}\hat{Q}^{\gamma_2}\hat{S}_2^{\eta_2} = k_{PQ}\hat{P}\hat{Q} + k_{SQ}\hat{S}_2\hat{Q} - K \tag{22}$$

Eliminating γ_2 and η_2 from the characteristic root equation (21) yields the quadratic equation:

$$Q(\beta_2) = \beta_2^2\{a\} + \beta_2\{b\} - \{c\} = 0 \tag{23}$$

$$a = \left\{\tfrac{1}{2}\sigma_P^2 - \rho_{PS}\sigma_P\sigma_S + \tfrac{1}{2}\sigma_S^2 + \frac{K^2}{2\hat{P}^2\hat{Q}^2k_{PQ}^2}[\sigma_S^2] + \frac{K}{\hat{P}\hat{Q}k_{PQ}}[\rho_{PS}\sigma_P\sigma_S - \sigma_S^2]\right\}$$

$$b = \left\{\theta_P - \theta_S - \tfrac{1}{2}\sigma_P^2 - \tfrac{1}{2}\sigma_S^2 + \rho_{PS}\sigma_P\sigma_S + \frac{K}{\hat{P}\hat{Q}k_{PQ}}[\theta_Q + \theta_S + \frac{\sigma_S^2}{2}]\right\}$$

$$c = -\left\{r - \theta_Q - \theta_S\right\}$$

The solution to this equation is again:

$$\beta_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{24}$$

The difference between (17) and (23) is that the Q volatility has been eliminated, but not the θ_Q.

Model 6

Stochastic Price and Quantity with a Permanent Deterministic Subsidy

We modify the analysis to consider the impact on the investment decision of a permanent deterministic government subsidy, denoted by S, but where the output Q and market price P are stochastic.

The PDE is:

$$\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_3}{\partial P^2} + \frac{1}{2}\sigma_Q^2 Q^2 \frac{\partial^2 F_3}{\partial Q^2} + PQ\rho_{PQ}\sigma_P\sigma_Q \frac{\partial^2 F_3}{\partial P \partial Q} + \theta_P P \frac{\partial F_3}{\partial P} + \theta_Q Q \frac{\partial F_3}{\partial Q} + \theta_S S \frac{\partial F_3}{\partial S} - rF_3 = 0. \qquad (25)$$

The solution to (25) is:

$$ROV_3 = F_3 = A_3 P^{\beta_3} Q^{\gamma_3} S^{\eta_3}. \qquad (26)$$

where β_3 , γ_3 and η_3 are the power parameters for this option value function. The parameters are linked through the characteristic root equation found by substituting (26) in (25):

$$\begin{aligned} Q(\beta_3, \gamma_3, \eta_3) &= \tfrac{1}{2}\sigma_P^2 \beta_3(\beta_3 - 1) + \tfrac{1}{2}\sigma_Q^2 \gamma_3(\gamma_3 - 1) + \\ &\rho_{PQ}\sigma_P\sigma_Q\beta_3\gamma_3 + \theta_P\beta_3 + \theta_Q\gamma_3 + \theta_S\eta_3 - r = 0 \quad . \end{aligned} \qquad (27)$$

Eliminating γ_3 and η_3 from the characteristic root equation yields the quadratic equation:

$$Q(\beta_3) = \beta_3^2\{a\} + \beta_3\{b\} - \{c\} = 0 \qquad (28)$$

$$a = \left\{ \tfrac{1}{2}\sigma_P^2 + \frac{K^2}{2\hat{P}^2\hat{Q}^2 k_{PQ}^2}[\sigma_Q^2] + \frac{K}{\hat{P}\hat{Q}k_{PQ}}[\rho_{PQ}\sigma_P\sigma_Q] \right\}$$

$$b = \{\theta_P - \theta_S - \tfrac{1}{2}\sigma_P^2 + \rho_{PQ}\sigma_P\sigma_Q + \frac{K}{\hat{P}\hat{Q}k_{PQ}}[\theta_Q + \theta_S + \frac{\sigma_Q^2}{2}]\}$$

$$c = -\left\{ r - \theta_Q - \theta_S \right\}$$

The solution to this equation is again: $$\beta_3 = \frac{-b+\sqrt{b^2-4ac}}{2a} \tag{29}$$

All of these models can easily be solved in Excel as shown in Figures 5, 6 and 7 below.

Figure 5

	A	B	C	D
1			**SUBSIDIES MODEL 4**	
2	INPUT	**Stochastic P & Q & S**		
3	P	22.5		
4	Q	10		
5	S	10	per kwh	
6	R	325	B3*B5+B4*B5	
7	K	4000		
8	σ_P	0.2		
9	σ_Q	0.2		
10	σ_S	0.2		
11	ρ_{PQ}	0		
12	ρ_{PS}	0		
13	ρ_{SQ}	0		
14	r	0.08		
15	θ_P	0.04		
16	θ_Q	0		
17	θ_S	0		
18	OUTPUT	**692.08**	R*	
19	a1	0.0550	0.5*(B8^2)+0.5*(B10^2)-B12*B8*B10+((B7^2)/(2*B34))*((B9^2)+2*B13*B9*B10+(B10^2))+B35	17
20	b1	0.0517	B15-B17-0.5*(B8^2)-0.5*(B10^2)+B11*B8*B9+B12*B8*B10-B13*B9*B10+B36	17
21	β1	0.8244	(-B20+SQRT((B20^2)-4*B19*(-B14+B16+B17+B13*B9*B10)))/(2*B19)	18
22	η1	1.2402	1+B21*((B7/(B28*B30*B29))-1)	16
23	γ1	2.0646	B21+B22	15
24	A1	0.0211	B33/(B21*(B28^B21)*(B29^B23)*(B25^B22))	12
25	**S^1**	**46.7077**	(B22*B28*B30)/(B21*B31)	13
26	**F1(P,Q,S)**	**555.5114**	IF(B5<B25, B24*(B3^B21)*(B4^B23)*(B5^B22),B27)	3
27	F1(P,Q,S)	3757.2233	(B30*B28*B29)+(B32*B25*B29)-B7	8
28	P^	22.5000		
29	Q^	10.0000		
30	P PV rP	13.7668	(1-EXP(-(B14-B15)*B38))/(B14-B15)	5
31	Q PV rQ	9.9763	(1-EXP(-(B14-B16)*B38))/(B14-B16)	6
32	S PV rS	9.9763	(1-EXP(-(B14-B17)*B38))/(B14-B17)	7
33	PQrPQ	3097.5246	B28*B29*B30	
34	P^2Q^1rPQ^2	9594658.5041	(B28^2)*(B29^2)*(B30^2)	
35	a2	-0.0517	(B7/B33)*(B11*B8*B9+B12*B8*B9-B13*B9*B10-(B10^2))	17
36	b2	0.0517	(B7/B33)*(B16+B17+0.5*(B9^2)+2*(B13*B9*B10)+0.5*(B10^2))	17
37	β1	0.8244	B33/(B33+B32*B25*B29-B7)	15
38	T	20.00000		
39	PDE	0.0000	0.5*(B8^2)*(B3^2)*B43+0.5*(B9^2)*(B4^2)*B44+0.5*(B10^2)*(B5^2)*B45+B15*B3*B40+B16*B4*B41+B17*B5*B42-B14*B26	2
40	ΔROV1,P	20.3544	B21*B24*(B3^(B21-1))*(B4^B23)*(B5^B22)	
41	ΔROV1,Q	114.6918	B23*B24*(B3^B21)*(B4^(B23-1))*(B5^B22)	
42	ΔROV1,S	68.8944	B22*B24*(B3^B21)*(B4^B23)*(B5^(B22-1))	
43	ΓROV1,P	-0.1588	B21*(B21-1)*B24*(B3^(B21-2))*(B4^B23)*(B5^B22)	
44	ΓROV1,Q	12.2103	B23*(B23-1)*B24*(B3^B21)*(B4^(B23-2))*(B5^B22)	
45	ΓROV1,S	1.6548	B22*(B22-1)*B24*(B3^B21)*(B4^B23)*(B5^(B22-2))	

Figure 6

	A	B	C	D
1			**SUBSIDIES MODEL 5**	
2	INPUT	**Stochastic P & S**		
3	P	22.50		
4	Q	10.00		
5	S	10.00	per kwh	
6	R	325.00	B3*B5+B4*B5	
7	K	4000.00		
8	σ_P	0.20		
9	σ_Q	**0.20**		
10	σ_S	0.20		
11	ρ_{PQ}	0.00		
12	ρ_{PS}	0.00		
13	ρ_{SQ}	0.00		
14	r	0.08		
15	θ_P	0.04		
16	θ_Q	0.00		
17	θ_S	0.00		
18	OUTPUT	**534.87**	R*	
19	a1	0.0217	0.5*(B8^2)+0.5*(B10^2)-B12*B8*B10+((B7^2)/(2*B34))*((B10^2))+B35	23
20	b1	0.0258	B15-B17-0.5*(B8^2)-0.5*(B10^2)+B12*B8*B10+B36	23
21	β2	1.4151	(-B20+SQRT((B20^2)-4*B19*(-B14+B16+B17)))/(2*B19)	24
22	η2	1.4123	1+B21*((B7/(B28*B30*B29))-1)	16
23	γ2	2.8274	B21+B22	14
24	A2	0.0003	B33/(B21*(B28^B21)*(B29^B23)*(B25^B22))	12
25	**S^2**	**30.9869**	(B22*B28*B30)/(B21*B31)	13
26	**F2(P,Q,S)**	**443.1266**	IF(B5<B25, B24*(B3^B21)*(B4^B23)*(B5^B22),B27)	20
27	F2(P,Q,S)	2188.8695	(B30*B28*B29)+(B32*B25*B29)-B7	22
28	P^	22.5000		
29	Q^	10.0000		
30	P PV rP	13.7668	(1-EXP(-(B14-B15)*20))/(B14-B15)	5
31	Q PV rQ	9.9763	(1-EXP(-(B14-B16)*20))/(B14-B16)	6
32	S PV rS	9.9763	(1-EXP(-(B14-B17)*20))/(B14-B17)	7
33	PQrPQ	3097.5246	B28*B29*B30	
34	P^2Q^1rPQ^2	9594658.5041	(B28^2)*(B29^2)*(B30^2)	
35	a2	-0.0517	(B7/B33)*(-(B10^2))	23
36	b2	0.0258	(B7/B33)*(B16+B17+0.5*(B10^2))	23
37	β2	1.4151	B33/(B33+B32*B25*B29-B7)	
38				
39	PDE	0.0000	0.5*(B8^2)*(B3^2)*B43+0.5*(B10^2)*(B5^2)*B45+B15*B3*B40+B16*B4*B41+B17*B5*B42-B14*B26	
40	ΔROV2,P	27.8702	B21*B24*(B3^(B21-1))*(B4^B23)*(B5^B22)	
41	ΔROV2,Q	125.2908	B23*B24*(B3^B21)*(B4^(B23-1))*(B5^B22)	
42	ΔROV2,S	62.5829	B22*B24*(B3^B21)*(B4^B23)*(B5^(B22-1))	
43	ΓROV2,P	0.5142	B21*(B21-1)*B24*(B3^(B21-2))*(B4^B23)*(B5^B22)	
44				
45	ΓROV2,S	2.5803	B22*(B22-1)*B24*(B3^B21)*(B4^B23)*(B5^(B22-2))	

These figures show an increased threshold over Models 1-2-3 with some of the same parameter values, because the facility is finite (20 years) rather than perpetual, although the investment opportunity is perpetual. Figure 5 shows a threshold of R*=692, with P,Q and S stochastic. Figure 6 shows a threshold of R*=534 with the same volatility for P and S, but Q is constant. Figure 7 shows R*=673 with a stochastic P and Q (since Q is volatile so is the extra revenue QS, even though S is assumed to be constant). If a government wants to encourage early investment though green certificate allocations, intervening in the certificate trading market to minimize volatility and drift, or an arrangement where the allocation of these certificates is inversely related to Q (which seems fair) would lower the threshold S that justifies immediate investment.

Figure 7

	A	B	C	D
1			**SUBSIDIES MODEL 6**	
2	INPUT	**Stochastic P & Q**		
3	P	22.50		
4	Q	10.00		
5	S	10.00	per kwh	
6	R	325.00	B3*B5+B4*B5	
7	K	4000.00		
8	σ_P	0.20		
9	σ_Q	0.20		
10	σs	**0.00**		
11	ρ_{PQ}	0.00		
12	ρ_{PS}	0.00		
13	ρ_{SQ}	0.00		
14	r	0.08		
15	θ_P	0.04		
16	θ_Q	0.00		
17	θs	0.00		
18	OUTPUT	**673.09**	R*	
19	a1	0.0534	0.5*(B8^2)+((B7^2)/(2*B34))*((B9^2))+B35	28
20	b1	0.0458	B15-B17-0.5*(B8^2)+B11*B8*B9+B36	28
21	β3	0.8682	(-B20+SQRT((B20^2)-4*B19*(-B14+B16+B17)))/(2*B19)	29
22	η3	1.2529	1+B21*((B7/(B28*B30*B29))-1)	16
23	γ3	2.1211	B21+B22	14
24	A3	0.0154	B33/(B21*(B28^B21)*(B29^B23)*(B25^B22))	12
25	**S^3**	**44.8092**	(B22*B28*B30)/(B21*B31)	13
26	**F3(P,Q,S)**	**544.8456**	IF(B5<B25, B24*(B3^B21)*(B4^B23)*(B5^B22),B27)	26
27	F3(P,Q,S)	3567.8196	(B30*B28*B29)+(B32*B25*B29)-B7	
28	P^	22.5000		
29	Q^	10.0000		
30	P PV rP	13.7668	(1-EXP(-(B14-B15)*20))/(B14-B15)	5
31	Q PV rQ	9.9763	(1-EXP(-(B14-B16)*20))/(B14-B16)	6
32	S PV rS	9.9763	(1-EXP(-(B14-B17)*20))/(B14-B17)	7
33	PQrPQ	3097.5246	B28*B29*B30	
34	P^2Q^1rPQ^2	9594658.5041	(B28^2)*(B29^2)*(B30^2)	
35	a2	0.0000	(B7/B33)*(B11*B8*B9+B12*B8*B9)	28
36	b2	0.0258	(B7/B33)*(B16+B17+0.5*(B9^2))	28
37	β3	0.8682	B33/(B33+B32*B25*B29-B7)	

14.3 Revenue Floors & Ceilings

The real American collar option for a certain asset confines the effective price within specified floor (lower) and ceiling (upper) limits. Acting as a risk moderator, the collar offers protection against the adversity from extreme falls in the output price or rises in the procurement price while simultaneously extracting some incremental value from favourable prices. Consequently, the upside gains partially compensate the downside losses. Unlike financial options, real American perpetuities are currently unobtainable from the market, but governments may be agreeable to grant and underwrite price limits in certain circumstances. The pursuance of an energy diversity goal may motivate governments to enact a policy that subsidizes renewable energy investors by guaranteeing a fixed price in the form of a contract-for-differences deal. Similarly, foreign investors are induced to locate in countries whose governments grant subsidized or preferential procurement prices for raw materials or energy. The role of these subsidies is to raise the investment option value and to reduce the investment threshold, which not only render an investment more attractive but also hasten its exercise.

In a real option framework there are several articles on the effect of a subsidy on the investment value and policy. Dixit (1991) studies price ceilings for regulated industries. Boomsma et al. (2012) evaluate energy subsidies. Barbosa et al. (2015) look at investment and tax subsidies, Adkins and Paxson (2014) consider permanent and retractable subsidies as do Boomsma and Linnerud (2015), but not revenue ceilings. Armada et al. (2012) investigate a subsidy in the form of a perpetual put option on the output price with protection against adverse price movements. None of these authors consider perpetual collar options. From our general model, separate price floor subsidies and price ceilings are specific examples of general collar options imposed on the active project value.

Here, output price gains are restricted to an upper ceiling limit so the firm is sacrificing upside potential. Consequently, a price collar option contributes both positively and negatively to the active project value. Eventually, we examine the impact of the collar option on the investment opportunity value and the threshold. The two collar elements produce distinctive effects. The first element arises from the presence of a floor limit, which makes the investment opportunity more attractive and leads to an earlier exercise. In contrast, the second element due to a price ceiling limit is only partially reflected in the investment opportunity. Although the presence of a price ceiling results in a fall in the investment option value, there is no impact at all on the investment threshold.

Fundamental Model

For a firm in a monopolistic situation confronting a single source of uncertainty due to price variability, the opportunity to invest in an irretrievable project at cost K depends on the price evolution, which is specified by:

$$\mathrm{d}P = \alpha P \mathrm{d}t + \sigma P \mathrm{d}W, \qquad (1)$$

where α denotes the expected price risk-neutral drift, σ the price volatility, and $\mathrm{d}W$ an increment of the standard Wiener process. Using contingent claims analysis, the option to invest in the project $F\left(P\right)$ follows the risk-neutral valuation relationship:

$$\tfrac{1}{2}\sigma^2 P^2 \frac{\partial^2 F}{\partial P^2} + \left(r-\delta\right) P \frac{\partial F}{\partial P} - rF = 0\,, \tag{2}$$

where $r > \alpha$ denotes the risk-free interest rate and $\delta = r - \alpha$ the rate of return shortfall. The generic solution to (2) is:

$$F\left(P\right) = A_1 P^{\beta_1} + A_2 P^{\beta_2}\,, \tag{3}$$

where A_1, A_2 are to be determined generic constants and β_1, β_2 are, respectively, the positive and negative roots of the fundamental equation, which are given by:

$$\beta_1, \beta_2 = \left(\tfrac{1}{2} - \frac{r-\delta}{\sigma^2}\right) \pm \sqrt{\left(\tfrac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}\ . \tag{4}$$

In (3), if $A_1 > 0$ then F is a continuously increasing function of P and represents an American perpetual call option, Samuelson (1965), while if $A_2 > 0$ then it is a decreasing function and represents a put option, Merton (1973).

Investment and Collar Option

The valuation of an active project with a collar is conceived over three mutually exclusive exhaustive regimes, I, II and III, defined on the P line, each with its own distinct valuation function. Regimes I, II and III are defined by $P \le P_L$, $P_L < P \le P_H$ and $P_H \le P$, respectively. We conjecture that the optimal price threshold $\hat{P}_C$ triggering an investment lies between the lower and upper collar price limits, $P_L \le \hat{P}_C \le P_H$.

If we can treat the optimal trigger price level as lying between P_L and P_H, then the optimal solution is obtainable from equating the investment option value with trigger level $P = \hat{P}_C$ with the value for an active project, , with $P = \hat{P}_C$ for $P_L \le P \le P_H$ net of the investment cost K. The solution is found when both the value-matching relationship:

$$A_0 P^{\beta_1} = \frac{PQ}{\delta} - A_1 P^{\beta_1} + A_2 P^{\beta_2} - K \tag{5}$$

and its smooth-pasting condition expressed as:

$$\beta_1 A_0 P^{\beta_1} = \frac{PQ}{\delta} - \beta_1 A_1 P^{\beta_1} + \beta_2 A_2 P^{\beta_2} \tag{6}$$

holds when evaluated for $P = \hat{P}_C$. This reveals:

$$\frac{\hat{P}_C Q}{\delta} = \frac{\beta_1}{\beta_1 - 1} K - \frac{\beta_1 - \beta_2}{\beta_1 - 1} A_2 \hat{P}_C^{\beta_1}, \tag{7}$$

$$A_0 = \frac{K \hat{P}_C^{-\beta_1}}{\beta_1 - 1} - \left(\frac{1 - \beta_2}{\beta_1 - 1} \right) A_2 \hat{P}_C^{\beta_2 - \beta_1} + A_1 . \tag{8}$$

Since the real collar model formulates the existence of both a floor and ceiling price, two distinct models, each representing the floor and ceiling price separately, can be derived from this general model.

The basic payoff diagram for a real collar is shown in Figure 8, where the asset value V ranges from 0 to 300, and the real call and put have the same exercise price K=150, and the same premiums=50.

Figure 8

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1						REAL OPTION PAYOFFS								
2	OWN ASSET	0	25	50	75	100	125	150	175	200	225	250	275	300
3	WRITE REAL CALL AT K2 INTRINSIC	50	50	50	50	50	50	50	50	25	0	-25	-50	-75
4	BUY REAL PUT AT K1 INTRINSIC	75	50	25	0	-25	-50	-50	-50	-50	-50	-50	-50	-50
5	STRATEGY RESULT	125	125	125	125	125	125	150	175	175	175	175	175	175
6	K2	175												
7	K1	125												
8	Debt, K													
9	Option cost, y2	50												
10	Option cost, y1	50												
11		Own asset subject to debt K, buy a "costless protective put" (=protective collar).												
12	**STRATEGY RESULT equals real collar.**													
13	WRITE REAL CALL AT K2 INTRINSIC	IF(B2>B6,B6-B2+B9,B9)												
14	BUY REAL PUT AT K1 INTRINSIC	IF(B2<B7,-B2-B7-B10,-B10)												
15	STRATEGY RESULT	B2+B3+B4												

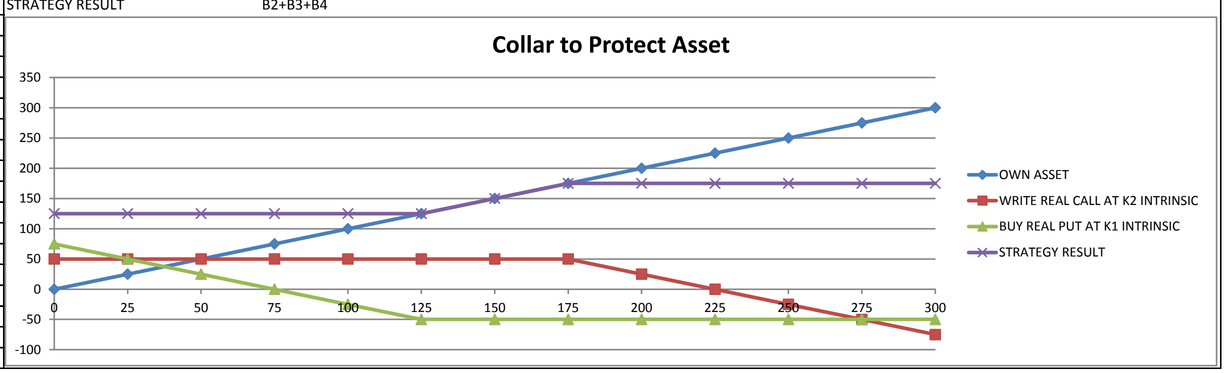
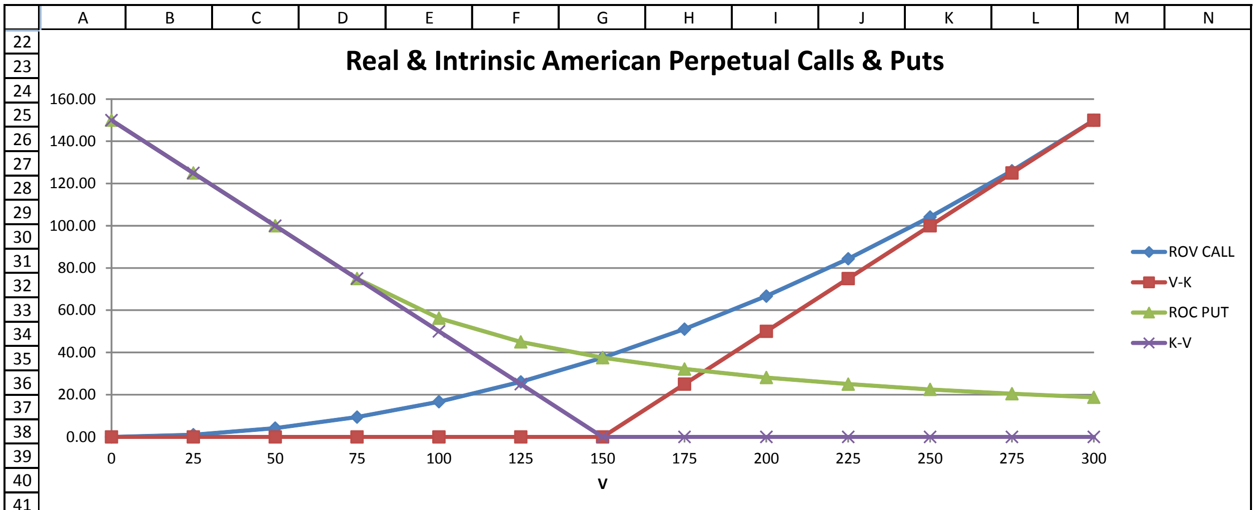


Figure 9 illustrates both the intrinsic (zero premium) and real call and put option on an asset V that ranges from 0 to 300. Note that at the extremes the call is equal to zero when V is zero, but the put is not zero even though V=300, so the real call and put values are asymmetric.

Figure 9



42	ROV CALL	IF(B2<B10,((B10-B3)*(B2/B10)^B11),B9)
43	V-K	MAX(B2-B3,0)
44	V*	(B11/(B11-1))*B3
45	β_1	0.5-(B5-B6)/(B4^2)+SQRT(((B5-B6)/(B4^2)-0.5)^2 + 2*B5/(B4^2))
46		
47	ROC PUT	IF(B2>B15,(-B3/(B16-1))*(B2/B15)^B16,B14)
48	K-V	MAX(B3-B2,0)
49	V*	(B16/(B16-1))*B3
50	β2	0.5-(B5-B6)/(B4^2)-SQRT(((B5-B6)/(B4^2)-0.5)^2 + 2*B5/(B4^2))

EXERCISE 14.1

Sonja believes she can build a solar plant for K=$4000 that will produce Q=10 KWh per year, that can be sold for P=$10 per KWh, P*Q=R. $ROV = B_1 R^{\beta_1}$, where β_1=2. For a subsidy τ, the threshold $\hat{R}$ that justifies immediate investment is: $\hat{R} = \frac{\beta_1}{\beta_1 - 1} K \frac{(r-\delta_R)}{(1+\tau)}, B_1 = \frac{(1+\tau)\hat{R}^{1-\beta_1}}{\beta_1(r-\delta_R)}$. If r=.07, electricity δ =.04, a proportional subsidy τ =1, should Sonja build now, or try to sell this opportunity for $2500?

PROBLEM 14.6

Sonja believes she can build a solar plant for K=4000 that will produce Q=10 KWh per year, that can be sold for P=22.25 per KWh, P*Q=R. $ROV = B_1 R^{\beta_1}$, where β_1 is the solution to a simple quadratic equation. For a proportional subsidy τ, the threshold $\hat{R}$ that justifies immediate investment is:

$$\hat{R} = \frac{\beta_1}{\beta_1 - 1} K \frac{(r - \delta_R)}{(1+\tau)}, B_1 = \frac{(1+\tau)\hat{R}^{1-\beta_1}}{\beta_1 (r - \delta_R)}, \beta_1 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{(\frac{r-\delta}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}}$$

If r=.08, electricity δ =.04, R volatility=.2, subsidy τ =.10, what R would justify immediate investment, and what is the value of this investment opportunity?

References

Abadie, L. and J. Chamorro. "Valuation of Wind Energy Projects: A Real Option Approach." Real Options Conference, Tokyo (2013), doi 10.3390/en 7053218.

Adkins, R., and D. Paxson. "Subsidies for Renewable Energy Facilities under Uncertainty" *The Manchester School* (2016), DOI 10.1111.

Adkins, R., and D. Paxson. "Analytical Investment Criteria for Subsidized Energy Facilities." *Real Options Conference Norway 2016*.

Adkins, R., and D. Paxson. "Real Collar Options." *Real Options Conference Norway 2016*.

Armada, M. J., P. J. Pereira, and A. Rodrigues. "Optimal subsidies and guarantees in public–private partnerships." *The European Journal of Finance* 18 (2012), 469-495.

Barbosa, D., V.M. Carvalho, and P.J. Pereira. "Public stimulus for private investment: An extended real options model". *Economic Modelling* xxx (2015).

Boomsma, T., N.Meade and S-E Fleten. "Renewable Energy Investments under Different Support Schemes: A Real Options Approach", *European Journal of Operational Research* 220 (2012), 225-237.

Boomsma, T., and K. Linnerud. "Market and Policy Risk under Different Renewable Electricity Support Schemes", *Energy* 89 (2015), 435-448.

Dixit. A. "Irreversible Investment with Price Ceilings". *Journal of Political Economy* (1991),541-557.

Merton. R.C. "Theory of Rational Option Pricing", *Bell Journal of Economics and Management Science* 4 (1973), 141-183.

Paxson, D., and H. Pinto. "Rivalry under Price and Quantity Uncertainty." *Review of Financial Economics*, 14 (2005), 209-224.

Ritzenhofen, I., and S. Spinler. “Optimal Design of Feed-in-Tariffs to Stimulate Renewable Energy Investments under Regulatory Uncertainty—A Real Options Analysis”, *Energy Economics* (2015).

Samuelson. P.A. “Rational Theory of Warrant Pricing". *Industrial Management Review* 6 (1965), 13-32.